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UTILIZATION OF THE BEND TEST FOR DETERMINING STRESS-STRAIN CURVES

JOHN CAMPO
MECHANICS OF MATERIALS DIVISION

July 1971

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13. ABSTRACT

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Technical Report by

JOHN CAMPO

July 1971

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ABSTRACT

Experimental verification of the utilization of the bend test for determining stress-strain curves for tension and compression is presented in this report. Stress-strain curves are determined from independently performed tension, compression, and bend tests of AISI 4340 steel specimens (heat treated to three nominal yield strength levels — 125,000, 175,000, and 225,000 psi, respectively, at 0.2% offset) and compared. Fair agreement is found to exist among the curves up to the 2% strain limit of each test.

CONTENTS

I. NOMENCLATURE

Bend Tests

a = moment arm or distance between either support and its closer load point (double point loading).

b,h = beam cross sectional dimensions, width and height, respectively.

P = applied load.

σ_1, σ_2 = absolute values of extreme fiber stresses, compression and tension, respectively.

ϵ_1, ϵ_2 = absolute values of corresponding extreme fiber strains.

δ = deflection at midspan of neutral axis from its original horizontal position.

Tension and Compression Tests

A_0 = original cross sectional area.

P = applied load.

$\sigma = P/A_0$ = absolute value of engineering stress.

ϵ = absolute value of axial ordinary strain.

II. INTRODUCTION

Interest in the bend test has undergone a revival in recent years because of the development and exploitation of such brittle materials as ceramics and carbides and because of the greater use of high-strength materials with little ductility.

The tension test has normally been one of the most common methods used for determining design data of a material. Sometimes, however, a tensile specimen may not be feasible either because of size limitations or because of difficulty in machining the particular material. The latter problem, usually with accompanying extraordinary expenses, is particularly encountered when dealing with such limited ductility materials as those mentioned above. In such cases, a bend test with its major advantage of employing a simple specimen with a rectangular cross section would be most welcome, provided that such a test could be used as a satisfactory substitute for the tension test.

Nadai,[1] in an analysis of the bending problem, provides the foundation for deriving the tensile and compressive stress-strain curves from the bend test. Bluhm[2] has modified the form of the resulting equation extending it to consideration of cases other than pure bending, illustrated its application, and suggested computational forms to facilitate the carrying out of the required operations.

In the present investigation, experimental verification of the proposed formulation is attempted. Tension, compression, and bend tests are independently performed on AISI 4340 steel specimens heat treated to three nominal yield-strength levels (125,000, 175,000, and 225,000 psi, respectively, at 0.2% offset). Stress-strain curves determined from the bend tests are then compared to those determined from the conventional tension and compression tests. The results tend to substantiate, at least up to the 2.0% strain limit of each test, that the proposed method is valid in obtaining a stress-strain curve for either tension or compression by utilization of the bend test.

Before continuing with the text, however, it should be brought out that: a) a steel, rather than some ceramic is chosen as the testing material in order to minimize machining and testing costs; b) an arbitrary value of 2.0% strain is selected as an upper test limit since many brittle materials will fracture below this strain value; and c) a computer program could obviously have been devised for automatic reduction of the test data.

III. VALUES OF STRESSES AND STRAINS FROM BEND TEST DATA*

Stresses

If bend tests are divided into three cases as noted below, and if, for ease in manipulation, the values of tensile and compressive stresses computed from the bend test data are expressed in absolute terms, that is, numerical values only, then the formulas presented by Bluhm become:

Case I. Different Stress-Strain Curves for Tension and Compression.

$$\sigma_1 = \left[\frac{a}{bh^2}\right]\left[P + \left(\frac{\epsilon_1+\epsilon_2}{2}\right)\left(\frac{dp}{d(\epsilon_1+\epsilon_2)}\right)\right]\left[\frac{d\epsilon_2}{d\epsilon_1} + 1\right] \quad (1)$$

$$\sigma_2 = \left[\frac{a}{bh^2}\right]\left[P + \left(\frac{\epsilon_1+\epsilon_2}{2}\right)\left(\frac{dp}{d(\epsilon_1+\epsilon_2)}\right)\right]\left[\frac{1}{\frac{d\epsilon_2}{d\epsilon_1}} + 1\right] \quad (2)$$

Case II. Identical Stress-Strain Curves for Tension and Compression.

$$\sigma_1 = \sigma_2 = \left[\frac{2a}{bh^2}\right]\left[P + \left(\frac{\epsilon}{2}\right)\left(\frac{dP}{d\epsilon}\right)\right] \quad (3)$$

Case III. Identical Stress Strain for Tension and Compression. Beam Restricted to Pure Bending.

$$\sigma_1 = \sigma_2 = \left[\frac{2a}{bh^2}\right]\left[P + \left(\frac{\delta}{2}\right)\left(\frac{d\delta}{d\epsilon}\right)\right] \quad (4)$$

---

*See Nomenclature (Section I) for definition of terms and Figure 1A for specimen configuration and loading arrangement.

Strains

The values of strains in the bend tests are simply those indicated by the strain gages, again in absolute terms.

Basic Consideration in Development of Formulas

Because this report is concerned chiefly with experimental verification of the expressions for determining tensile and/or compressive stress-strain curves from bend test data (Eqs. (1) through (4), inclusive), the actual development of the formulas, to be found in Reference [illegible], is intentionally omitted. However, the basic considerations involved in the development are listed here:

1. An initially straight beam of constant cross section is loaded either by forces perpendicular to its longitudinal axis in one of the principal planes of inertia of the cross section or by moments in one of the planes.

2. The cross-sectional dimensions of the beam are small in comparison to its length so that shear deformations may be ignored.

3. Cross sections of the beam perpendicular to the axis remain plane during bending.

4. For a beam restricted to pure bending, it is implied that the whole beam bends into a circular arc. For the more general loading condition, however, it is necessary that only the portion of the beam between two closely-spaced cross sections under consideration (along the length of the strain gage, for example) bend approximately into a circular arc.

IV. VALUES OF STRESSES AND STRAINS FROM CONVENTIONAL TENSION AND COMPRESSION TEST DATA*

Stresses and Strains

Since the total strain in any one test does not exceed [illegible], and since the test results are for comparison purposes only, it is deemed sufficient to determine engineering stress-ordinary strain rather than true stress-true strain curves. Consequently, the value of stress is simply that obtained by dividing the applied load by the original cross-sectional area of the specimen, and the value of strain, that obtained by averaging the values indicated by the strain gages, again in absolute terms.

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*See Nomenclature (Section I) for definition of terms and Figures [illegible] and [illegible] for specimen configuration and loading arrangements.

V. SELECTION OF SPECIMEN SIZES

Bend Specimen

More than one experimenter has found that varying the size of test specimens leads to different bend test properties. From the data presented by Grobe and Roberts[3] (when employing the double-point loading system) it may be seen that higher bend-strength values are obtained from a 2-in. rather than a 4-in. span (distance between outermost supports) or from a 0.250-in. rather than a 0.500-in. beam width. Burghardt and Elder[4] remind us to maintain a reasonable span (they chose 0.500-in.) and keep the h/a ratio, that is, ratio of beam height to moment arm*, smaller than 2/3 in order to prevent shear stresses from becoming too pronounced. Duckworth[5] also suggests that the h/a ratio be kept as small as practicable to avoid pronounced shear stresses, and, in addition, that the b/h ratio, that is, ratio of beam width to beam height, be kept reasonably small to avoid stiffening effect due to relatively wide beams. Because of the above findings and because of space requirements for the application of strain gages and their leads, a 3-in. span was judged to be satisfactory, and a bar with a 3 1/4-in. overall length, a 0.600-in. width, and a 0.150-in. height was chosen as the bend specimen.

Tension Specimen

A threaded tensile specimen with a 0.357-in. uniform diameter was used in this investigation.

Compression Specimen

According to ASTM EN-61,[6] a compression specimen should be round, and, if of medium length, should have a length-to-diameter ratio of 3.0. A 5/8-in. diameter was selected in order to keep the applied load from exceeding the 120,000 pounds capacity of the testing machine and, yet, to have ample space along the specimen for strain-gage application. In compliance with the recommended length-to-diameter ratio of 3.0, the specimen was made 2.0 in. long.

VI. SPECIMEN PREPARATION

Material

AISI 4340 steel was chosen for the experimental work in this investigation. In order to insure uniform material properties, all specimens were taken from one plate, 12 x 12 x 3/4 in., and the length of each specimen was always in the longitudinal direction of the plate. In addition, the width of each bend specimen was taken in the transverse direction of the plate.

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*See Figure 1A for explanation of moment arm, a.

Heat Treatment

Roughly cut blanks, all with the original 3/4-in. thickness were heat treated before machining, as indicated below, to three nominal yield strength values (125,000, 175,000, and 225,000 psi, respectively, at 0.2% offset). All blanks were normalized by heating at 1650°F for one hr and by air cooling. They were then austenitized at 1550°F for one hr and oil quenched. Double tempering followed at the temperatures and for the periods of time indicated:

Level 1 - nominal yield strength of 125,000 psi - 1200°F (1+1) hr.

Level 2 - nominal yield strength of 175,000 psi - 950°F (1/2+1/2) hr.

Level 3 - nominal yield strength of 225,000 psi - 400°F (2+2) hr.

Tempering was performed in salt baths.

Surface Grinding

After heat treatment, the bend specimens were brought to within 0.010 or 0.015 in. of the final dimensions by rough grinding and were then finished by fine surface grinding. A similar procedure was followed for the ends of the compression specimens.

Number of Specimens

Four tension, 4 compression, and 5 bend specimens in each of the three nominal yield-strength levels, or a total of 39 specimens in all, were tested.

VII. MEASURING DEVICES AND EXPERIMENTAL PROCEDURES

Bend Tests

Since strain gage readings on both the top and bottom surfaces were required, double-point loading (see Figure 1A) was desirable. Longitudinal strain gages (SR4, type FAE-25-PL) were mounted, one each of mid-span on both the top and bottom surfaces of each specimen. The strain gages were electrically connected to separate X-Y recorders so that simultaneous readings could be made.

The testing was performed on a Tinius Olsen testing machine, manually operated at head speeds well within the limits recommended in ASTM standards.[6] Strain-rate effects were considered to be negligible. A signal from a pressure transducer in the testing machine, indicating the applied load, was also connected to the X-Y recorders, and, in this way, plots of load vs strain were autographically recorded. Each test was continued until a total strain of 2.0% was indicated. Figure 2 shows the test setup.

In this setup, load and support points consisted of "rollers" that were free to rotate in order to minimize friction effects.

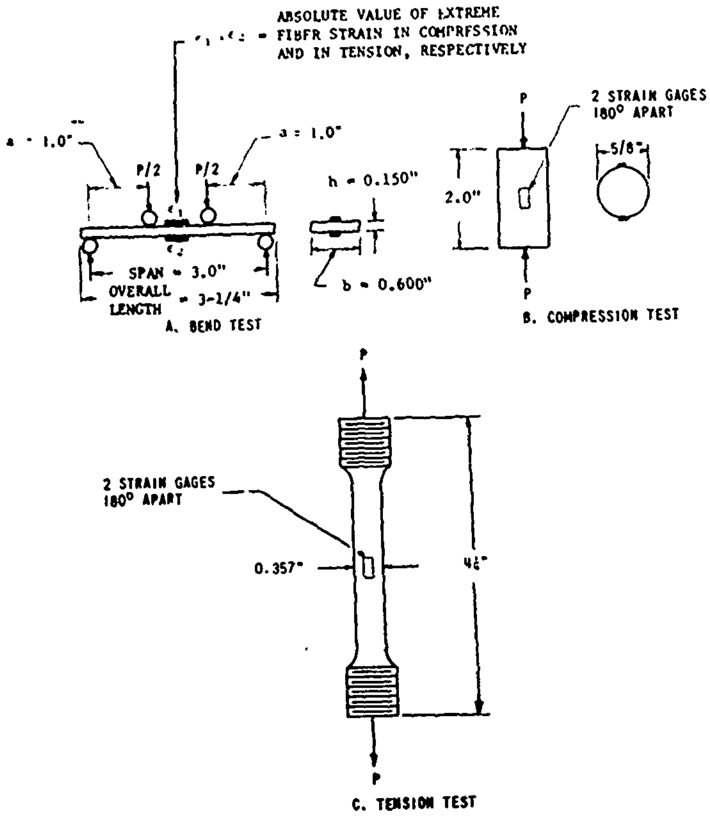

Figure 1. Sketch of loading arrangement for bend, compression, and tension test specimens

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Figure 2. Bend test setup (A), employing "friction free" fixture (B), showing specimen under load. Double point loading. 0.150 x 0.600 x 3.25-inch specimen

Tension and Compression Tests

Since axial ordinary strains of the specimens were required in these tests, longitudinal strain gages (SR4, type PFA-50-12-L) were mounted at midspan and 180° apart (see Figures 1B and 1C). These gages were connected to an X-Y recorder so that average strain only could be recorded.

The testing was also performed on a Tinius Olsen testing machine, and, again, the head speeds were kept well within the limits recommended in ASTM standards. The load signal of the testing machine was also connected to the X-Y recorder, but the recorder was calibrated so that engineering stress (rather than applied load) could be indicated directly. For the tension and compression tests, then, plots of engineering stress vs strain were autographically recorded. Again, each test was continued until a total strain of 2.0% was reached. Figure 3 illustrates the testing arrangement for tension tests and Figure 4 that for compression tests.

VIII. RESULTS AND DISCUSSION

Recorded Data

Results were generally obtained in the form of X-Y recordings. These recordings consisted of applied load vs extreme fiber strains for the bend tests and engineering stress-strain curves, directly, for the tensile and compressive tests.

Auxiliary Curves for Bend Tests

When the extreme fiber strains of a bend test specimen were not equal, two auxiliary curves had to be constructed before any stress-strain curve could be determined. These auxiliary curves consisted of two plots, one of applied load vs sum of extreme fiber strains and one of extreme fiber strain in compression vs extreme fiber strain in tension. No such auxiliary curve was required when the extreme fiber strains were equal.

Stress-Strain Curves for Each Specimen

The originally obtained X-Y recordings in the tensile and compressive tests consisted of engineering stress-strain curves, directly, and no further treatment of the data was required in determining these results. For the bend tests, however, to determine the required stress-strain curves, Eqs. (1) and (2) of the text along with slope data of the auxiliary curves were employed when the extreme fiber strains were not equal, and Eq. (3) of the text along with slope data of either of the originally obtained X-Y recordings were employed when the extreme fiber strains were equal.

Stress-Strain Curves for Each Yield Strength Level

In addition, because the stress-strain curves were essentially identical for any one strength level and for any one type of testing, stress-strain curves for each yield strength level were determined by averaging the individual test

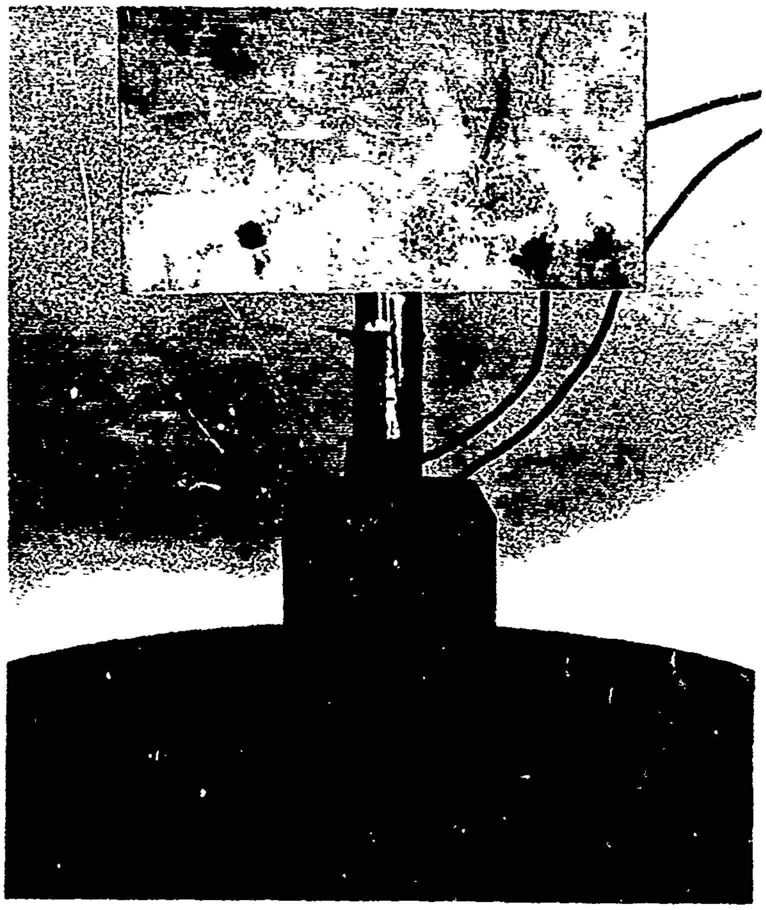

Figure 4. Compression test setup showing specimen under load. 0.625-inch diameter x 2.0-inch specimen
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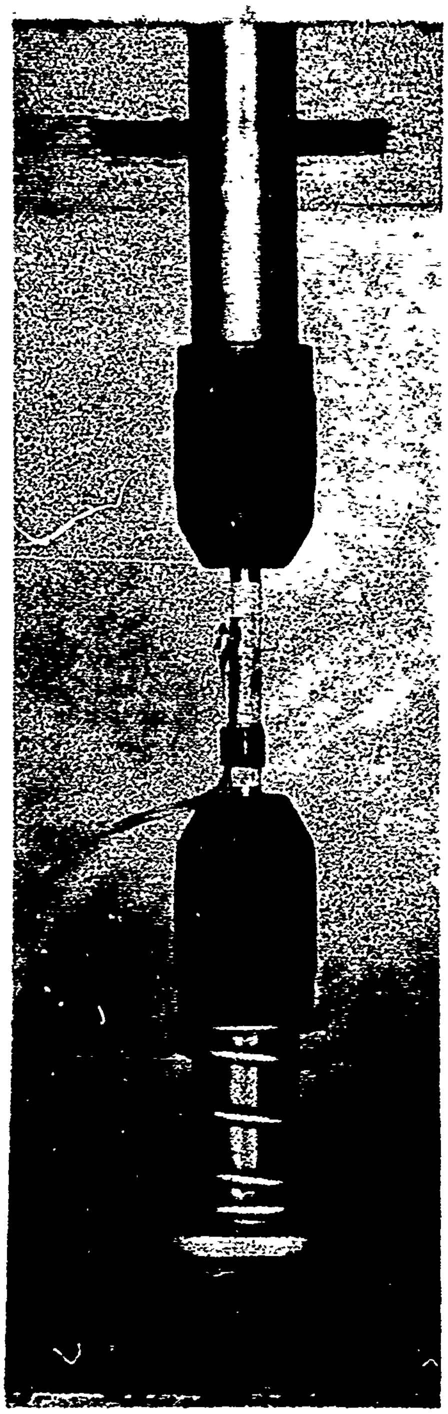

Figure 3. Tension test setup showing specimen under load. 0.357-inch uniform diameter x 4.25-inch threaded type specimen
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results. It was felt that a comparison of the stress-strain curves based on the average test results would be less bulky and just as indicative as a comparison of the curves based on the individual test results themselves.

Reported Results

The average test data and corresponding values of stresses and strains for all three nominal yield-strength level specimens are summarized in Tables I through IV. Stress-strain curves are given in Figure 5 where the plotted values are taken from the aforementioned tables. In addition, pertinent values of mechanical properties (including values of Young's moduli, yield strength at 0.2% offset and flow stresses at both 1.0% and 2.0% of total strain - all determined from the stress-strain curves of Figure 5) are listed in Tables V through VII. Maximum percentage scatter and percentage discrepancies (both these terms are subsequently defined and discussed) are also included in these tables.

Auxiliary curves, when required, and stress-strain curves, both in tension and in compression, have been constructed for each bend specimen as well as for each yield strength level. As typical examples, however, only those curves for yield strength level 1 are presented in Figures 6, 7, and 8. In fact, only the stress-strain curve in compression is depicted in the last figure.

Scatter

If scatter is defined as the amount that an individual test value differs from the average test value of a particular number of specimens tested, then

$$\%\ \text{scatter} = \frac{\text{individual test value - average test value}}{\text{average test value}} \times 100.$$

Scatter is expected to be greater in stress-strain curves derived from bend tests than those derived from conventional tests. This is true because, as described above, in conventional tests, stress-strain curves may be obtained directly, whereas, in bend tests, slope data must first be determined either from one of the originally obtained X-Y recordings or from auxiliary constructed curves. The work involved in obtaining the required slope data introduces more leeway for experimental error and, hence, greater scatter.

Discrepancy

If discrepancy is defined as the amount that a bend test value differs from the corresponding conventional test value, then

$$\%\ \text{discrepancy} = \frac{\text{bend test value - conventional test value}}{\text{conventional test value}} \times 100.$$

Discrepancy values, then, indicate the degree of agreement, or rather, of disagreement, among results predicted by bend tests and those determined by conventional tests.

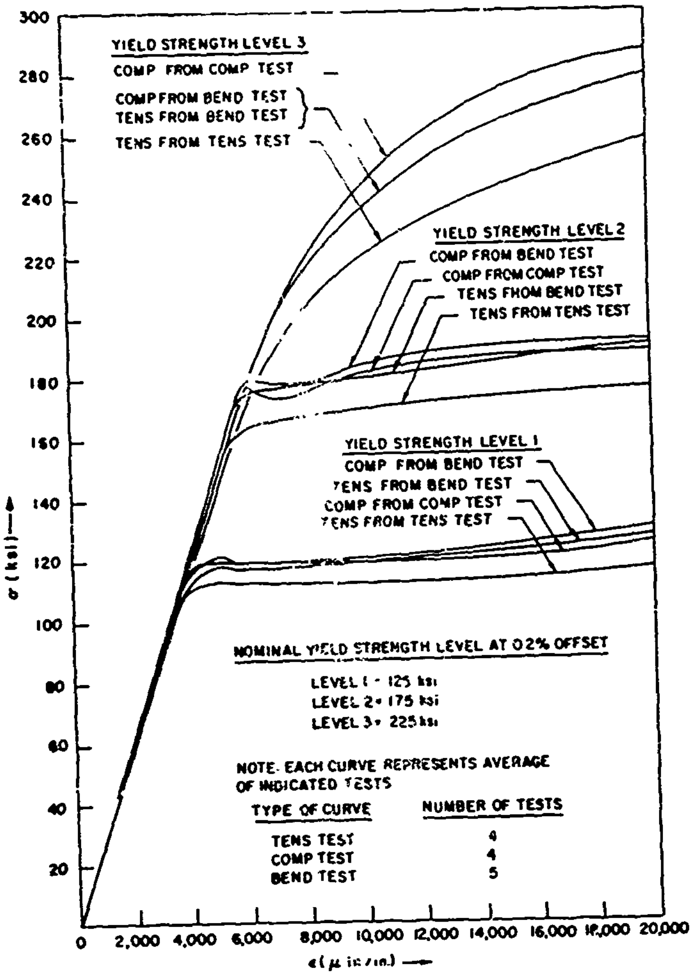

Figure 5. Stress-strain curves from bend, tension, and compression tests

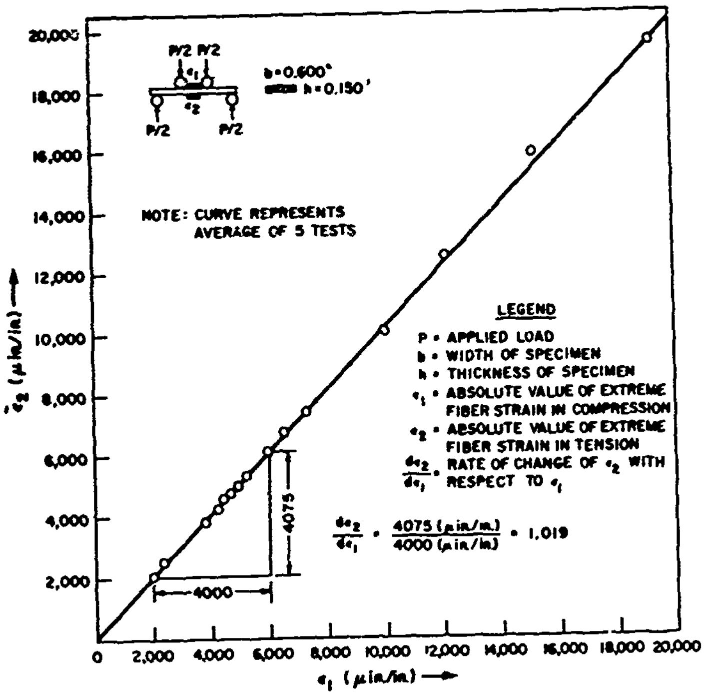

Figure 6. Plot of tensile strain vs compressive strain for yield strength level 1 bend specimens (auxiliary curve)

Figure 7. Plot of load vs sum of strains for yield strength level 1 bend specimens (auxiliary curve)

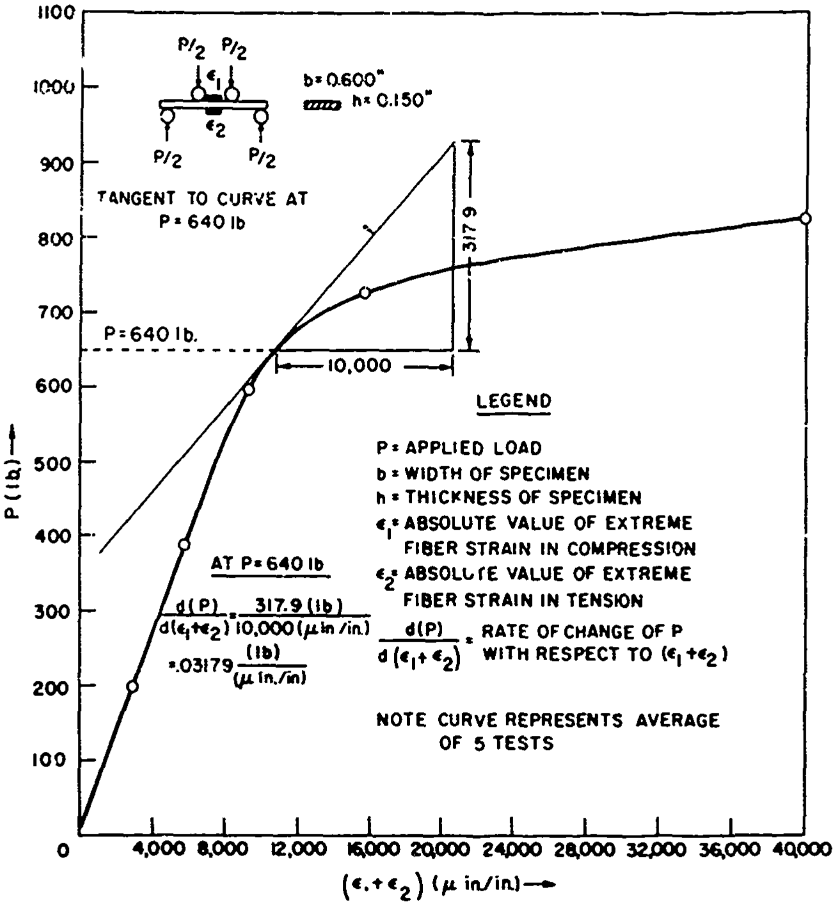

Figure 8. Plot of stress vs strain in compression for yield strength level 1 bend specimens

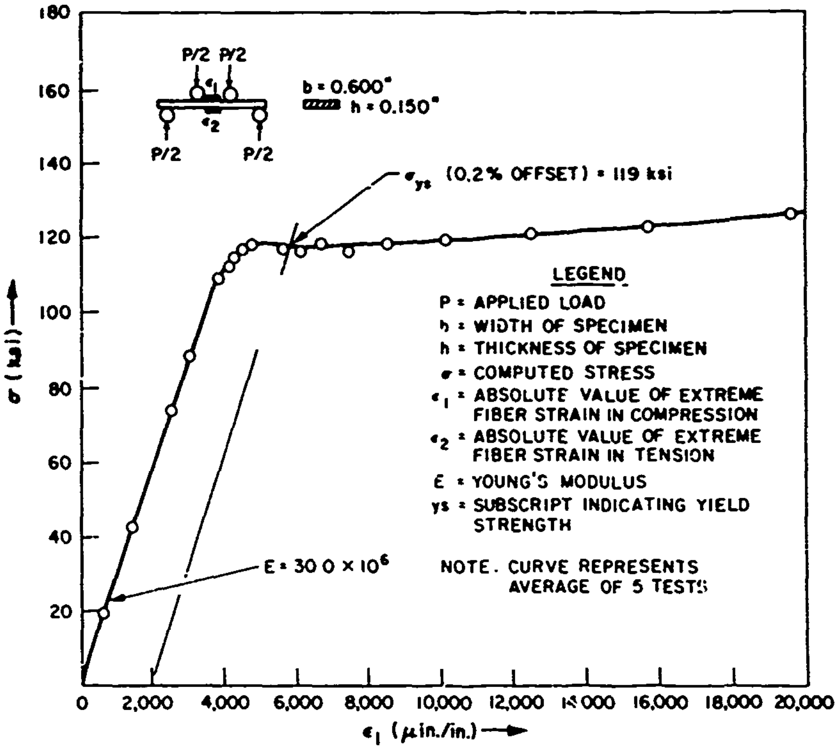

Table I. AVERAGE STRESS-STRAIN VALUES FROM YIELD-STRENGTH LEVEL 1 BEND SPECIMENS

① P (lb)	② ϵ_1 ←	③ ϵ_2 (μin/in)	④ $\frac{\epsilon_1+\epsilon_2}{2}$ →	⑤ $\frac{dP}{d(\epsilon_1+\epsilon_2)}$ (lb/μin/in)	⑥ σ_1 (lb/in^2)	⑦ σ_2 (lb/in^2)
0	0	0	0	0.06614	0	0
100	749	763	756	0.06614	22,433	22,015
200	1498	1526	1512	0.06614	44,866	44,030
300	2247	2289	2268	0.06614	67,300	66,045
400	2996	3052	3024	0.06614	89,733	88,060
500	3745	3815	3780	0.06614	112,167	110,075
520	3881	3953	3917	0.06063	113,287	111,174
540	4055	4131	4093	0.05423	113,957	111,832
560	4240	4319	4280	0.05107	116,441	114,270
580	4433	4515	4474	0.04850	119,194	116,972
600	4657	4745	4701	0.04325	120,141	117,901
620	4927	5019	4973	0.03800	120,986	118,730
640	5196	5294	5254	0.03179	120,652	118,403
660	5553	5657	5605	0.02370	118,574	116,363
680	6004	6116	6060	0.01808	118,083	115,881
700	6608	6732	6670	0.01550	120,151	117,911
720	7292	7428	7360	0.00962	118,268	116,063
740	8372	8528	8450	0.00732	119,897	117,662
760	9947	10,133	10,040	0.00507	121,274	119,013
780	12,242	12,470	12,356	0.00392	123,898	121,159
800	15,406	15,694	15,550	0.00287	126,319	123,964
820	19,260	19,620	19,440	0.00245	129,759	127,339

NOTES:

1) Circled numbers identify appropriate columns

2) $$\textcircled{6} = \left[\frac{a}{bh^2}\right]\left[\textcircled{1} + \textcircled{4} \times \textcircled{5}\right]\left[\frac{d\epsilon_2}{d\epsilon_1} + 1\right] \quad \text{Eq. (1)}$$

3) $$\textcircled{7} = \left[\frac{a}{bh^2}\right]\left[\textcircled{1} + \textcircled{4} \times \textcircled{5}\right]\left[\frac{1}{\frac{d\epsilon_2}{d\epsilon_1}} + 1\right] \quad \text{Eq. (2)}$$

4) $$\frac{a}{bh^2} = \frac{1.0}{(.600)(.150^2)} = 74.074$$

5) $$\frac{d\epsilon_2}{d\epsilon_1} = 1.019$$

6) Above values represent averages of five tests

Table II. AVERAGE STRESS-STRAIN VALUES FROM YIELD-STRENGTH LEVEL 2 BEND SPECIMENS

① P (lb)	② ϵ_1 ←	③ ϵ_2 (μin/in)	④ $\frac{\epsilon_1+\epsilon_2}{2}$ →	⑤ $\frac{dP}{d(\epsilon_1+\epsilon_2)}$ (lb/μin/in)	⑥ σ_1 (lb/in^2)	⑦ σ_2 (lb/in^2)
0	0	0	0	0.06717	0	0
200	1478	1500	1489	0.06717	44,778	44,116
400	2955	3000	2978	0.06717	89,555	88,232
600	4433	4500	4467	0.06717	134,333	132,348
800	5911	5999	5955	0.06717	179,111	176,464
820	6052	6142	6097	0.06098	177,886	175,257
840	6213	6306	6260	0.05917	180,663	177,993
860	6396	6492	6444	0.05181	178,194	175,561
880	6604	6703	6654	0.04608	177,114	174,496
900	6898	7002	6950	0.03788	173,629	171,063
920	7084	7191	7138	0.03623	175,918	173,319
940	7320	7430	7375	0.03367	177,368	174,746
960	7668	7783	7726	0.02494	172,049	169,507
980	8015	8135	8075	0.02778	179,756	177,099
1000	8402	8528	8465	0.02370	179,203	176,555
1020	8769	8901	8835	0.02110	180,069	177,408
1040	9281	9420	9351	0.01930	182,166	179,474
1060	9851	9999	9925	0.01720	183,695	180,980
1080	10,484	10,642	10,563	0.01475	184,454	181,728
1100	11,232	11,400	11,316	0.01330	186,649	183,890
1120	12,010	12,190	12,100	0.01180	188,481	185,696
1140	12,854	13,046	12,950	0.00975	189,001	186,208
1160	13,995	14,205	14,100	0.00780	189,556	186,755
1180	15,533	15,767	15,650	0.00580	189,674	186,871
1200	17,866	18,134	18,000	0.00400	189,858	187,052
1210	18,821	19,104	18,763	0.00330	189,944	187,137
1220	20,261	20,564	20,413	0.00260	190,017	187,209

NOTES:

1) Circled numbers identify appropriate columns

2) $$⑥ = \left[\frac{a}{bh^2}\right]\left[① + ④ \times ⑤\right]\left[\frac{d\epsilon_2}{d\epsilon_1} + 1\right] \quad \text{Eq. (1)}$$

3) $$⑦ = \left[\frac{a}{bh^2}\right]\left[① + ④ \times ⑤\right]\left[1 \Big/ \frac{d\epsilon_2}{d\epsilon_1} + 1\right] \quad \text{Eq. (2)}$$

4) $\frac{a}{bh^2} = \frac{1.0}{(.600)(.150^2)} = 74.074$

5) $\frac{d\epsilon_2}{d\epsilon_1} = 1.015$

6) Above values represent averages of five tests

Table III. AVERAGE STRESS-STRAIN VALUES FROM YIELD-STRENGTH LEVEL 3 BEND SPECIMENS

① P (lb)	② $\epsilon_1 = \epsilon_2 = \epsilon$ (μin/in)	③ $\frac{dP}{d\epsilon}$ (lb/μin/in)	④ $\sigma_1 = \sigma_2 = \sigma$ (lb/in^2)
0	0	0.13098	0
200	1527	0.13098	44,444
400	3054	0.13098	88,889
600	4581	0.13098	133,333
800	6108	0.13098	177,778
840	6414	0.12880	185,639
880	6734	0.12640	193,420
920	7055	0.12120	199,634
960	7402	0.11000	202,535
1000	7769	0.10440	208,228
1040	8171	0.10080	215,084
1080	8554	0.09470	220,005
1120	8991	0.08960	225,5[illegible]
1160	9446	0.08060	228,248
1200	9942	0.07600	233,747
1240	10,490	0.07250	240,659
1280	11,068	0.06950	246,669
1320	11,607	0.06100	248,003
1360	12,358	0.05560	252,373
1400	13,114	0.04890	254,905
1440	13,948	0.04450	259,5[illegible]
1480	14,936	0.04050	264,067
1520	15,875	0.03680	265,459
1560	16,992	0.03240	271,852
1580	17,542	0.03030	273,446
1600	18,364	0.02830	275,114
1620	19,034	0.02640	277,222
1640	19,792	0.02460	279,028

NOTES:

1) Circled numbers identify appropriate columns

2) $④ = \left[\frac{2a}{bh^2}\right]\left[① + \frac{② \times ③}{2}\right]$ Eq. (5)

3) $\frac{2a}{bh^2} = \frac{(2)(1.0)}{(.600)(.150^2)} = 148,148$

4) Above values represent averages of five tests

Table 11. [illegible]

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NOTES:

1. [illegible]

2. [illegible]

3. [illegible]

Table V. PERTINENT MECHANICAL PROPERTIES OF YIELD-STRENGTH LEVEL 1 SPECIMENS INCLUDING MAXIMUM PERCENTAGE SCATTER AND PERCENTAGE DISCREPANCIES

Property in Tension	Determined from Tension Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	29.4×10^6	+0.3 -0.2	28.9×10^6	+0.7 -2.6	-1.7
Yield Strength at 0.2% Offset	112,750	+1.1 -0.7	117,000	+1.7 -2.6	+3.8
Flow Stress at 1.0% Strain	112,750	+1.5 -0.3	118,900	+0.5 -2.0	+5.5
Flow Stress at 2.0% Strain	116,000	0	127,400	+0.5 -1.1	+9.8

Property in Compression	Determined from Compression Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	30.0×10^6	+1.1 -2.2	30.0×10^6	+0.9 -3.4	0
Yield Strength at 0.2% Offset	119,000	+1.4 -0.8	119,000	+0.4 -1.7	0
Flow Stress at 1.0% Strain	119,500	+1.0 -0.9	121,300	+0.2 -2.7	+1.5
Flow Stress at 2.0% Strain	125,100	+1.0 -0.9	130,200	+0.6 -3.2	+4.1

NOTES:

1) $\% \text{ Scatter} = \dfrac{\text{Individual Test Value} - \text{Average Test Value}}{\text{Average Test Value}} \times 100$

2) $\% \text{ Discrepancy} = \dfrac{\text{Bend Test Value} - \text{Conventional Test Value}}{\text{Conventional Test Value}} \times 100$

3) Above values represent averages of four tests in tension, four tests in compression, and five tests in bending

Table VI. PERTINENT MECHANICAL PROPERTIES OF YIELD-STRENGTH LEVEL 2 SPECIMENS INCLUDING MAXIMUM PERCENTAGE SCATTER AND PERCENTAGE DISCREPANCIES

Property in Tension	Determined from Tension Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	29.1×10^6	+1.5 -0.2	29.4×10^6	+0.3 -0.5	+1.0
Yield Strength at 0.2% Offset	166,000	+0.9 -0.1	175,000	+3.9 -1.1	+5.4
Flow Stress at 1.0% Strain	169,900	+0.5 -0.2	181,000	+1.1 -2.3	+6.5
Flow Stress at 2.0% Strain	175,600	+1.4 -0.9	187,200	+2.0 -0.2	+6.6

Property in Compression	Determined from Compression Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	30.4×10^6	+1.5 -1.2	30.3×10^6	+2.0 -3.5	-0.3
Yield Strength at 0.2% Offset	177,000	+0.4 -1.0	178,000	+0.7 -1.1	+0.6
Flow Stress at 1.0% Strain	179,900	+0.6 -1.1	183,800	+0.2 -2.1	+2.2
Flow Stress at 2.0% Strain	188,600	+0.6 -1.0	190,000	+2.1 -0.2	+0.7

NOTES:

1) $\%\ \text{Scatter} = \dfrac{\text{Individual Test Value} - \text{Average Test Value}}{\text{Average Test Value}} \times 100$

2) $\%\ \text{Discrepancy} = \dfrac{\text{Bend Test Value} - \text{Conventional Test Value}}{\text{Conventional Test Value}} \times 100$

3) Above values represent averages of four tests in tension, four tests in compression, and five tests in bending

Table VII. PERTINENT MECHANICAL PROPERTIES OF YIELD-STRENGTH LEVEL 3 SPECIMENS INCLUDING MAXIMUM PERCENTAGE SCATTER AND PERCENTAGE DISCREPANCIES

Property in Tension	Determined from Tension Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	28.9×10^6	+1.5 -1.4	29.1×10^6	+0.9 -3.1	+0.7
Yield Strength at 0.2% Offset	214,000	+0.6 -0.8	232,000	+3.0 -1.5	+8.4
Flow Stress at 1.0% Strain	218,500	+1.1 -0.7	234,500	+1.9 -1.6	+7.3
Flow Stress at 2.0% Strain	257,350	+1.0 -1.5	279,500	+1.3 -1.8	+8.6

Property in Compression	Determined from Compression Test (psi)	Maximum Scatter (%)	Determined from Bend Test (psi)	Maximum Scatter (%)	Discrepancy (%)
Young's Modulus	29.8×10^6	+1.3 -0.2	29.1×10^6	+1.8 -3.8	-2.4
Yield Strength at 0.2% Offset	242,000	+0.5 -1.0	232,000	+1.0 -2.4	-4.1
Flow Stress at 1.0% Strain	240,750	+0.7 -0.6	234,500	+0.9 -1.7	-2.6
Flow Stress at 2.0% Strain	287,200	+0.6 -0.8	279,500	+1.4 -1.6	-2.7

NOTES:

1) $\% \text{ Scatter} = \dfrac{\text{Individual Test Value - Average Test Value}}{\text{Average Test Value}} \times 100$

2) $\% \text{ Discrepancy} = \dfrac{\text{Bend Test Value - Conventional Test Value}}{\text{Conventional Test Value}} \times 100$

3) Above values represent averages of four tests in tension, four tests in compression, and five tests in bending

Acceptable Limits of Maximum Scatter

An experimental error analysis was made based on errors of ± 1.0% in strain gage indications, ± 0.5% in load indications, ± 0.001 in. in cross-sectional dimensions, and ± 0.025 in. in X-Y chart readings. According to this analysis, experimental errors could be as high as ± 1.5% in conventional tests and ± 4.0% in bend tests. These values of maximum expected experimental errors, then, are the acceptable limits of maximum scatter.

Comparison of Stress-Strain Curves

From Figure 5, it may be seen that stress-strain curves determined from bend tests agree fairly well with those determined from conventional tests. Also from Figure 5, it may be noted from the conventional tests that, at any strain level, the corresponding values of stress in compression are generally higher than those in tension. This same trend is brought out by the bend tests.

Observations concerning both scatter and discrepancies are best noted from the results reported in Tables V through VII.

Scatter, as expected, was generally higher in bend tests than in conventional tests. The acceptable limits of maximum-percentage scatter were never exceeded in the bend tests and only once in the conventional tests — in the determination of Young's modulus in compression for yield strength level 1 specimens. Even in this case, however, scatter was considered acceptable not only because of the low value of scatter determined (-2.2%) but also because of the limited number of tests involved (four). Maximum percentage scatter, then, was generally within the computed allowable limits, and, therefore scatter was not deemed to be objectionable in any of the testing.

Discrepancy values, as mentioned earlier, are measures of agreement among results predicted by the bend tests and those determined by the conventional tests.

Very close agreement was generally found among the values of Young's moduli predicted by the bend tests and those determined by the conventional tests. The largest value of discrepancy of -2.4% occurred for the value of Young's modulus in compression for yield strength level 3 specimens. Very good agreement, then, within 2.4%, was found in the elastic portions of the stress-strain curves predicted by the bend tests and those determined by the conventional tests.

Beyond the elastic region, on the other hand, at least within the 2.0% of total strain to which all specimens were subjected, the flow stress values predicted by the bend tests were generally higher than those determined by the conventional tests. The largest value of discrepancy of +9.8% occurred for the value of flow stress in tension at 2.0% of total strain for yield-strength level 1 specimens. Fair agreement, then, (within 9.8%) was found to exist in the early plastic portions of the stress-strain curves predicted by the bend tests and those determined by the conventional tests.

IX. SUMMARY AND CONCLUSIONS

1. Expressions from Nadai[1], adapted for experimental exploitation, have been presented that permit the calculation of the early portion of the stress-strain curve, both in tension and in compression, from bend tests. These expressions are applicable to beams, whether or not subjected to pure bending, and are particularly adaptable to materials which fail with relatively small deformations. They are limited to the extent that shear stresses in the bend specimen must be negligible.

2. Three of these expressions (two of which are applicable when the stress-strain curve in tension differs from that in compression and one of which is applicable when the stress-strain curves in tension and in compression are equal) have been applied to bend specimens of steel, heat treated to three different yield-strength levels. The bend tests were carried out to strain values of 2.0%.

3. Independent tension and compression tests were performed on specimens made from the same stock and subjected to the same heat treatment as the bend specimens. These conventional tests were also carried out to 2.0% strain.

4. Stress-strain curves determined from the bend, tension, and compression tests have also been presented. Mechanical properties derived from these curves, including Young's moduli, yield strengths at 0.2% offset, and flow stresses at both 1.0% and 2.0% total strain have also been presented. Maximum percentage scatter and percentage discrepancies have been included with these tabulated values.

5. Finally, a comparison of the results predicted by the bend tests and those determined by the tension and compression tests of the specimens, at least within the first 2.0% strain, indicated that:

a. Scatter was generally higher in bend tests than in conventional tests, but not objectionable.

b. For the present tests, the observation from conventional tests that, at any strain level, the corresponding value of stress in compression was generally higher than that in tension was confirmed in the bend-test results.

c. Also for the present tests, the results indicated that the bend test could be regarded as a very good substitute for the conventional tests (within 2.4% agreement) for determining the elastic portions of stress-strain curves and a fair substitute (within 9.8% agreement) for determining the early plastic portions of stress-strain curves.

ACKNOWLEDGMENTS

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